1. (a) $S(t) = -16t^2 + 96t + 256$

$v(t) = -32t + 96$

$a(t) = -32$

(b) $v(t) = 0$

$-32t + 96 = 0$

$-32t = -96$

$\boxed{t = 3 \text{ sec}}$

$+ \quad | \quad -$ $\quad v(t)$

$\quad\; 3$

(c) $S(3) = -16(3)^2 + 96(3) + 256$

$= -144 + 288 + 256$

$\boxed{= 400 \text{ ft}}$

(d) $S(t) = 0$

$-16t^2 + 96t + 256 = 0$

$-16(t^2 - 6t - 16) = 0$

$-16(t+2)(t-8) = 0$

$t = -2 \quad \boxed{t = 8 \text{ sec}}$

$t = -2$ is not in the domain for time.

(e) $V(8) = -32(8) + 96$

$= -256 + 96$

$= -160 \text{ ft/sec}$

(f) $S(0) = 256$

$S(3) = 400$

$S(5) = 336$

$\left.\begin{matrix} S(0) \\ S(3) \end{matrix}\right\} 144 \qquad \left.\begin{matrix} S(3) \\ S(5) \end{matrix}\right\} 64$

Total distance $= 144 + 64 = \boxed{208 \text{ ft}}$

(g) $S(0) - S(5)$

$256 - 336 =$

$\boxed{-80 \text{ ft}}$

(h) $S(t) = 144$

$-16t^2 + 96t + 256 = 144$

$-16t^2 + 96t + 112 = 0$

$-16(t^2 - 6t - 7) = 0$

$-16(t+1)(t-7) = 0$

$t = -1 \quad \boxed{t = 7}$

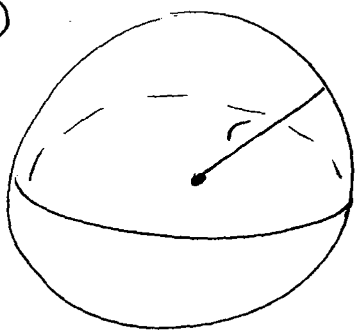
$t = -1$ is not in domain

$V(7) = -32(7) + 96$

$-244 + 96$

$\boxed{-128 \text{ ft/sec}}$

②



A) $\frac{dV}{dt} = 500$

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$r = 30$

$500 = 4\pi (30)^2 \frac{dr}{dt}$

$$\boxed{\frac{dr}{dt} = \frac{5}{36\pi} \approx .0442 \text{ cm/min}}$$

B) $\frac{dV}{dt} = 500$

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$r = 60$

$500 = 4\pi (60)^2 \frac{dr}{dt}$

$$\boxed{\frac{dr}{dt} = \frac{5}{144\pi} \approx .0111 \text{ cm/min}}$$

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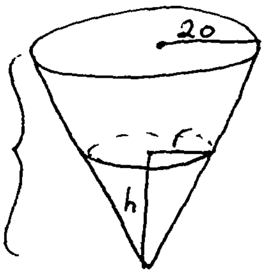
③



$\frac{dV}{dt} = -5$

$V = \frac{1}{3}\pi r^2 h$

$h = 7$

$V = \frac{1}{3}\pi \left(\frac{10}{7}h\right)^2 h$

$\frac{dh}{dt} = ?$

$V = \frac{100}{147}\pi h^3$

$\frac{20}{r} = \frac{14}{h}$

$\frac{dV}{dt} = \frac{100}{49}\pi h^2 \frac{dh}{dt}$

$20h = 14r$

$-5 = \frac{100}{49}\pi (7)^2 \frac{dh}{dt}$

$r = \frac{10}{7}h$

$$\boxed{\frac{dh}{dt} = -\frac{1}{20\pi} \approx -.0159 \text{ ft/sec}}$$

④ $f(x) = x^3 - 3x^2 - 24x + 2 \quad [-4, 8]$

ⓐ $f'(x) = 3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

$3(x+2)(x-4) = 0$

$x = -2 \quad x = 4$ → $f(-2) = 30$

$f(4) = -78$

$f'(x)$: $+$ on $(-\infty, -2)$, $-$ on $(-2, 4)$, $+$ on $(4, \infty)$

$\boxed{\begin{array}{l}\text{ans: } (-2, 30) \text{ is a relative max} \\ (4, -78) \text{ is a relative min}\end{array}}$

ⓑ $f(-4) = -14$

$f(-2) = 30$

$f(4) = -78 \leftarrow$

$f(8) = 130 \leftarrow$

$\boxed{\begin{array}{l}\text{abs min: } (4, -78) \\ \text{abs max: } (8, 130)\end{array}}$

ⓒ Inc: $(-\infty, -2) \cup (4, \infty)$ ⓓ Dec: $(-2, 4)$

ⓔ $f''(x) = 6x - 6 = 0$

$x = 1$

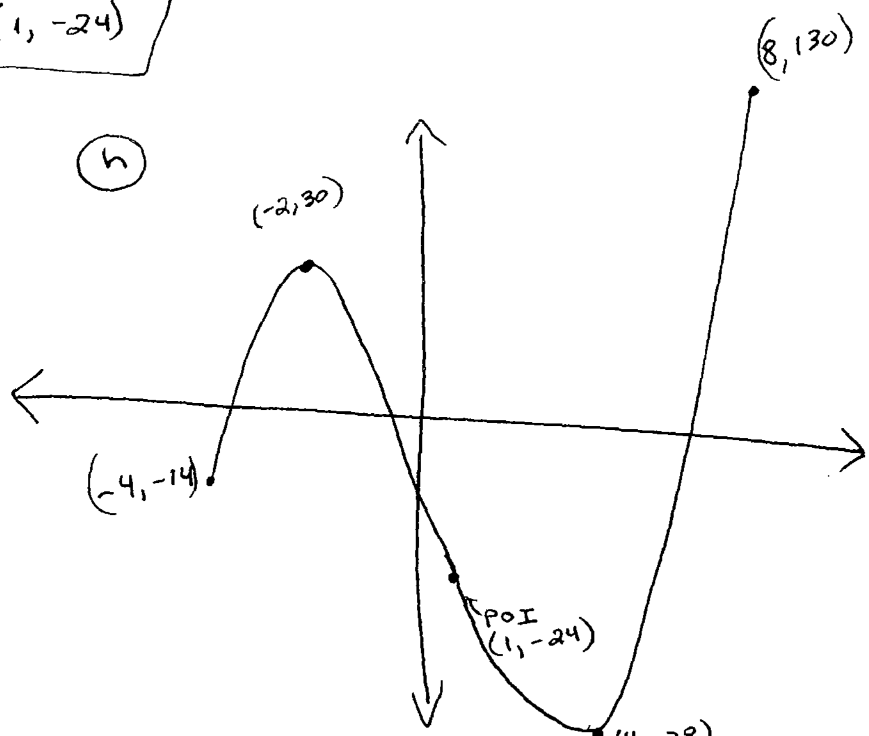
$f'(x)$: $-$ on $(-\infty, 1)$, $+$ on $(1, \infty)$

$f(1) = -24 \rightarrow \boxed{\text{P.O.I. at } (1, -24)}$

ⓕ concave up: $(1, \infty)$

ⓖ concave down: $(-\infty, 1)$

ⓗ



⑤ $f(x) = (x+2)^{2/3}$ $[-3, 6]$

ⓐ $f'(x) = \frac{2}{3}(x+2)^{-\frac{1}{3}} = 0$ or undefined

$= \frac{2}{3\sqrt[3]{x+2}}$ critical value at $x=-2$

$f'(x)$: $-$ on $(-\infty, -2)$, $+$ on $(-2, \infty)$

$f(-2) = 0 \rightarrow$ $\boxed{\text{relative min at } (-2, 0)}$

ⓑ $f(-3) = 1$
$f(-2) = 0$
$f(6) = 4$

$\boxed{\begin{array}{l}\text{absolute min } (-2, 0)\\ \text{absolute max } (6, 4)\end{array}}$

ⓒ Inc: $(-2, \infty)$ ⓓ Dec: $(-\infty, -2)$

ⓔ $f''(x) = -\frac{2}{9}(x+2)^{-\frac{4}{3}} = 0$ or undef

critical value at $x=-2$

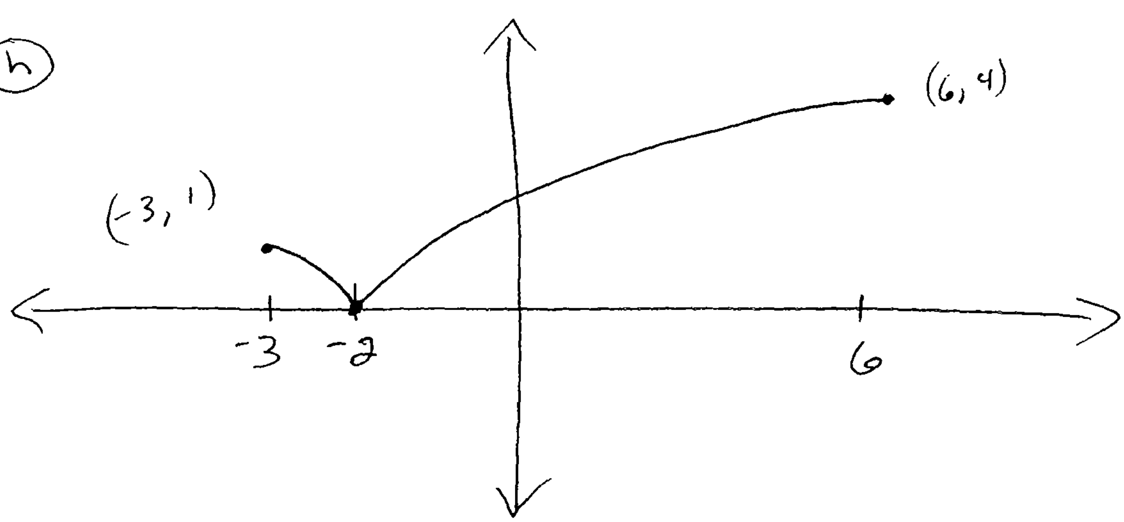
$f''(x)$: $-$ on $(-\infty, -2)$, $-$ on $(-2, \infty)$

$\boxed{\text{No P.O.I.}}$

ⓕ concave up: (none)

ⓖ concave down: $(-\infty, -2) \cup (-2, \infty)$

ⓗ



⑥ $f(x) = e^{\frac{x}{2}}$ $[-1, 4]$

$f(-1) = .6065$ $\quad m = \dfrac{7.389 - .6065}{4 + 1} = \dfrac{6.7825}{5} \approx 1.3565$

$f(4) = 7.389$

$f'(x) = 1.3565$

$e^{\frac{x}{2}}\left(\frac{1}{2}\right) = 1.3565$

$\boxed{X = 1.9961}$

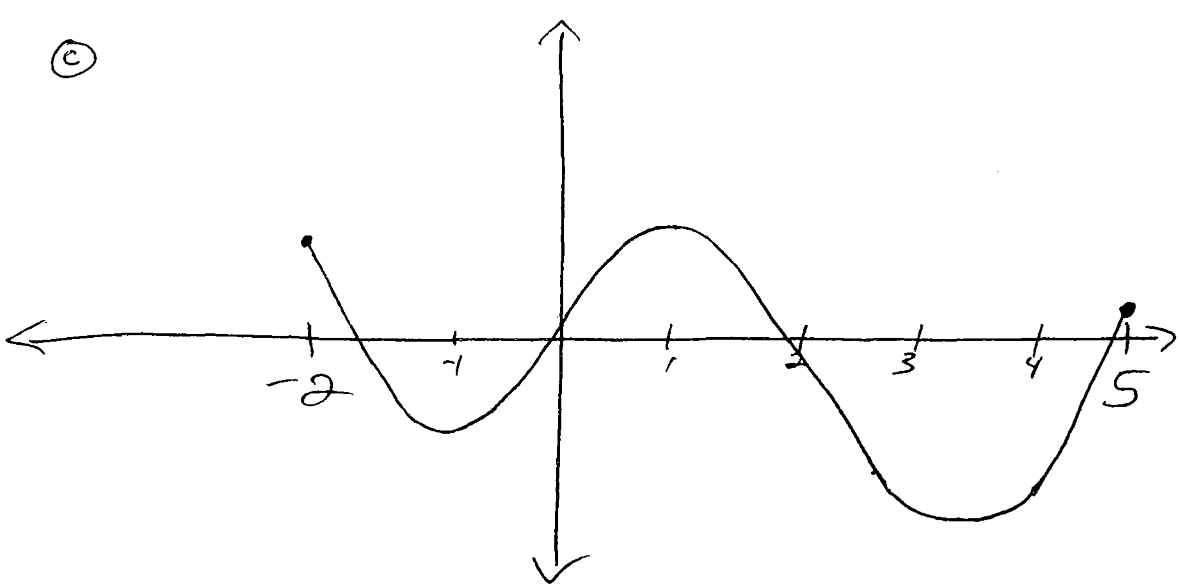
⑦ ⓐ - at $X = -1$ there is a min. The values of $f'(x)$ change from negative to positive.

- at $X = 1$ ther is a max. the values of $f'(x)$ change from positive to negative.

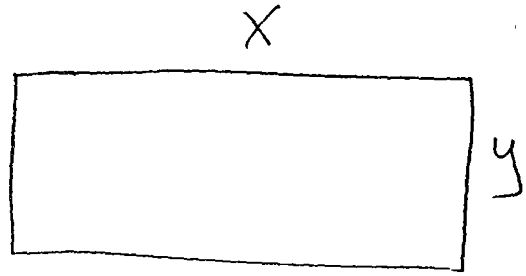
- at $X = 4$ there is a min. the values of $f'(x)$ changes from negative to positive.

ⓑ at $X = 0$ and $X = 3$ $f(x)$ has points of inflection. P.O.I.s occur when $f''(x) = 0$. That means that this graph must have a horizontal tangent.

ⓒ



(8)



Primary Eq:

$A = x\,y$

$A = x\left(-\frac{2}{3}x + 1250\right)$

$A = -\frac{2}{3}x^2 + 1250x$

Restrictions:

$2(x)(4) + 2(y)(6) = 15000$

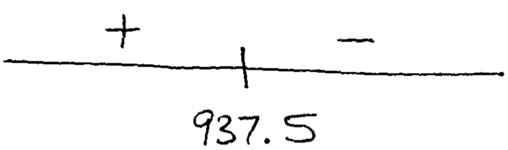
$8x + 12y = 15000$

$y = -\frac{2}{3}x + 1250$

$\frac{dA}{dx} = -\frac{4}{3}x + 1250 = 0$

$-\frac{4}{3}x = -1250$

$x = 937.5 \longrightarrow y = \frac{-2}{3}(937.5) + 1250$



$y = 625$

$\boxed{937.5 \text{ ft} \times 625 \text{ ft}}$